$\hat{(\cdot)}$ = estimate of $(\cdot)$
$(\cdot)^*$ = optimal value of $(\cdot)$
$\overline{(\cdot)}$ = mean value of $(\cdot)$

Subscripts

e = experimental value
d = calculated value

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Particle Turbulent Diffusion in a Dust Laden Round Jet

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The turbulent diffusion mechanism of particles in a round air jet are studied both theoretically and experimentally, with particular attention to the relative velocity between particles and fluid, overshooting effect of particles, and the distributions of fluid properties in space. The results indicate that the particle diffusivity decreases with the increase of the particle inertia. In general, the turbulent diffusivity of particles in an air jet is smaller than that of fluid scalar quantities. The particle inertia and the fluid large eddies, which are expressed by the Stokes number and the integral scale, respectively, play an important role in the transport mechanism of particles.

SCOPE

Attention has been focused on the diffusion of small particles in dust laden turbulent jets. Such a phenomenon is of interest in rocket engine exhausts, atomized fuel injection systems, spraying and waste disposal plumes, and also applied to such diverse applications as numerous cleaning devices and aerosol production. The principal purpose of the present study is to reveal the mechanism of the particle turbulent diffusion in the dust laden jet by predicting particle turbulent diffusivities theoretically and measuring them experimentally.

Many theoretical investigations of the turbulent diffusion of particles in the stream have been done. Most of the work, however, paid less attention to the effects on the particle diffusion summed up as follows.

1. The effect of the overshooting of particles from one eddy to another.
2. The effect of the relative velocity of particles to fluid due to the interaction between fluid movement and particles.
3. The effect of the various distributions of time averaged fluid velocities and turbulent properties (intensity, time scale, etc.) in space.

Moreover, in experimental works the assumption was also made that time averaged velocities of particles and fluids are equal, so that the turbulent diffusivity of particles may be calculated from the experimental result of concentration distribution.

In order to describe the mechanism of the turbulent diffusion of particles correctly, the factors described above should be taken into consideration, especially in a real situation, such as a jet flow.

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Hinze (1959) derived the diffusivity of particles in a homogeneous turbulence, which does not have the distribution of time averaged velocities and intensities, by solving the Lagrangian equation of particles. He assumed that the same fluid will surround the particle as it moves (no overshooting). The mechanism of a real turbulence is such that it is hardly possible for this to be satisfied. Accordingly, his result indicated that the turbulent diffusivities of particles and fluid become equal when t (diffusion time) $>> T_L$ (integral time scale), which obviously contradicts to the actual phenomenon.

Bullin and Dukler (1974) proposed modeling the turbulent gas diffusion process by repeated solutions of the Langevin equation on a hybrid computer. In their approach, velocity fluctuations were simulated by using white noise filtered in such a way as to reproduce the spectrum and the root-mean-square of the turbulent fluctuations. Recently, Lee and Dukler (1976) extended the method of Bullin and Dukler to include the presence of turbulent shear stress. This has been done by generating input fluctuating signals correlated with each other. They, however, did not treat a particle turbulent diffusion process.

In this work, we calculate the turbulent diffusivity of particles by considering the distributions of time averaged fluid velocities and turbulent properties in space, the overshooting effect of particles from one eddy to another, and the relative velocity of particles to fluid, as follows. First, the turbulent Lagrangian trajectories of particles are calculated by solving the Lagrangian equation of particles directly through a digital computer. Second, the displacements of many particles started from the same point are mean squared. Since this equation is a second-order, nonlinear, stochastic equation, it is very difficult to calculate it in consideration of all the eddies of fluid. Accordingly, we calculate it paying attention to the behavior of the energy containing eddy which is approximately expressed by a diffusion scale (Lagrangian integral scale).

The distribution of particle concentrations in a jet is measured by the change of particle turbulent properties, and then the turbulent diffusivity of particles is obtained from the experimental values in almost the same way as Van der Hegge Zijnen (1958). This is compared and discussed with the calculated results.

The effect of the turbulent properties of fluid and particles on the turbulent diffusion is explained in detail.

CONCLUSIONS AND SIGNIFICANCE

Calculated results based on our model and experimental results of the turbulent diffusivities were in good agreement. The results indicate that the turbulent diffusivity of particles in a jet is not equal to that of a fluid, even when the diffusion time $t >>$ the Lagrangian integral time scale T_L, and that the former is smaller than the latter. Accordingly, Hinze's equation cannot be applied to the particle diffusion in a turbulent jet. When one attempts to describe the mechanism of the turbulent diffusion of particles, the most important factors are the Stokes number of particles and the distribution of T_L, the intensity and time averaged velocities of fluid. The turbulent diffusivity of fine particles is predominantly characterized only by the Stokes number when the flow field is identical. Then, the larger the Stokes number or particle inertia, the smaller the particle turbulent diffusivity.

There is the region where time averaged particle velocity U_p is not equal to time averaged air velocity U in the dust laden jet. But U_p becomes equal to U in the region far from the jet exit (that is, $z \geqq 30D$ in the case of $\Psi = 3.4$, $z \geqq 42D$ in the case of $\Psi = 15$, and $z \geqq 65D$ in the case of $\Psi = 30$). The particle diffusivity in a turbulent round jet increases with z initially and eventually becomes nearly constant, and the distance from the jet exit where the diffusivity reaches constant coincides with the distance where U_p becomes equal to U.

This simple method of simulating turbulent diffusivity with mean velocity, intensity, and integral time scale distributions will permit the prediction of particle turbulent diffusion in free turbulent streams and boundary layers.

Buossinesq (1877) introduced a turbulent diffusivity by analogy to the molecular diffusion term $D_B\ \partial C/\partial x_i$, such that

$$- <C'u_i'> = \epsilon_i \frac{\partial C}{\partial x_i} \quad (1)$$

The turbulent diffusion equation, which is derived from the time averaged conservation of mass and Equation (1), takes the form

$$C\nabla \cdot U_p + U_p \cdot \nabla C = \nabla \cdot \epsilon \nabla C \quad (2)$$

The basic problem now is to derive an expression for ϵ. Taylor (1921) presented the first attempt at a description of the diffusivity of turbulence. In a turbulent field which is homogeneous in space and stationary in time, Taylor found the dependence of the Lagrangian mean dispersion of a fluid particle with time for limiting cases of small and large dispersion times. Bachelor (1949, 1952) showed that the dispersion depends on the velocity correlation of a single particle and that in homogeneous turbulence, the probability distribution of the displacement of the particle always tends to Gaussian distribution. Batchelor (1957) that noted when the probability distribution of the displacement of particles has a Gaussian form, the diffusivity can be interpreted as follows:

$$\epsilon_{pr} = \frac{1}{2} \frac{d}{dt} < (r_p - <r_p>)^2> \quad (3)$$

He also reported that the diffusivity initially increases with time and eventually becomes constant.

Corrsin (1953) reported the effect of a gradient of the mean velocity on turbulent diffusion by considering a flow field with a uniform velocity gradient, where the correlation between cross velocity components is zero. Riley and Corrsin (1971) simulated the dispersion for homogeneous turbulent shear flows by using a digital computer. Their results were in good agreement with the long and short diffusion time prediction of Corrsin's approach mentioned above.

Lee and Dukler (1976) proposed modeling the turbulent diffusion with the turbulent shear stress by repeated solutions of the Langevin equation. They found that diffusion in the presence of both gradient and shear is shown to be independent of the degree of correlation between velocities for diffusion time $t \geqq 2\,500$ s or $t \leqq 100$ s, but that diffusion is significantly influenced by the existence of cross correlation at all intermediate times. All of these studies, however, are on the diffusion of fluid lumps. The diffusion of solid or liquid particles with much larger density than that of fluid, have received very little attention.

Tchen (1947) derived the Lagrangian equation of motion for particles suspended in an unsteady velocity field by extending the Basset-Buossinesq-Oseen equation. Corrsin and Lumley (1956) improved the pressure gradient term in Tchen's equation. Hinze (1959) further suggested considering small particles and limiting the discussion to locally uniform fields and simplified this equation. The equation for the i component of the flow in the Stokes law regime takes the form

$$\frac{dv_{pi}}{dt} + \frac{36\mu}{(2\rho_p + \rho)d^2}(v_{pi} - v_i) = \frac{3\rho}{2\rho_p + \rho}\frac{dv_i}{dt} + \frac{18}{(2\rho_p + \rho)d}\sqrt{\frac{\rho\mu}{\pi}}\int_{t_0}^{t}\frac{\dfrac{dv_i}{dt'} - \dfrac{dv_{pi}}{dt'}}{\sqrt{t - t'}}\,dt' \quad (4)$$

Hinze (1959) analyzed particle movements in the turbulent flow, whose intensity and time averaged velocity are constant, by using Equation (4) and provided the relation between the particle diffusivity ϵ_p and the fluid lump diffusivity ϵ_f. In the analysis, he assumed the neighborhood that during the motion of the particle was formed by the same fluid. The result indicates that ϵ_p and ϵ_f become equal when the diffusion time $t >> $ the fluid integral time scale T_L. This is a questionable result, because we can not avoid the possibility that the particle will escape from its originally surrounding fluid.

Peskin (1959, 1962) has studied this problem. He assumes a joint Gaussian distribution for the fluid velocities, in which case the expected velocity encountered by the discrete particles may be equal to the Langrangian velocity times the spatial Eulerian correlation coefficient corresponding to the distance h between the centroids of the originally surrounding fluid and the particle. Peskin restricts his calculation to small values of h. Hence, the result is a slightly reduced diffusivity of the particles relative to the fluid. Recently, Davidson and McComb (1975) calculated the particle diffusivity in a round aerosol jet by using Hinze's relation. It is not adequate to use Hinze's relation for the particle diffusion in a jet. Other approximate solutions of simplified physical models for various flows are those of Kantrowitz (1940), Soo and Peskin (1958), and Kuboi et al. (1974). Their results, although fruitful in some cases, have been rather limited. On the other hand, many experimental studies of the eddy diffusion in a jet flow have been also done. Van der Hegge Zijnen (1958) pointed out that scalar quantity, too, satisfies the similarity in the developing region of a jet flow and that it tends to diffuse more easily than the momentum of fluid, through the measurement of the distribution of scalars (temperature and gas concentration) in both a round jet flow and a two-dimensional one. Recently, measuring the turbulent diffusion quantity of Helium gas in a round et, Aihara et al. (1974) found that the behavior of a large eddy plays an important role in the turbulent diffusion. Bashir and Uberoi (1975) have shown that the turbulent intensity of scalar, as well as that of fluid velocity, satisfies the similarity in the developing region of a two-dimensional jet by measuring experimentally the turbulent intensity of temperature. Goldschmidt and Eskinazi (1966) examined the distribution of concentrations of the liquid particles of the average diameter on a weight basis 3.3 μm in a two-dimensional jet. They indicated that the particle mass tends not to diffuse more easily than the fluid momentum, but only after calculating diffusivities by the diffusion equation, using the experimental results of the concentration distribution in the same method as Van der Hegge Zijnen (1958).

In the case of derivation of diffusivities, they have made assumptions that the average velocities of particles and fluid are equal and that the diffusivity at each cross section is constant. Although such a region where the assumptions are reasonable is considered to exist, the assumptions are not applied to the whole region of a jet. The region where those assumptions can be applied should be estimated quantitatively. Hedman and Smoot (1975) indicated through measurement of the concentration distribution of aluminum particles of diameters 6 and 30 μm that the mass of particles tends not to diffuse more easily than the fluid momentum. In the present work, a particle diffusion mechanism is analyzed with the Lagrangian and the Eulerian considerations.

JET FLOW

Values of the time averaged velocity, the fluctuation velocity, and the integral scale of fluid are required to calculate particle velocities, trajectories, and diffusivities in a jet. Values used for the present calculation are shown in the following section.

Time Averaged Fluid Velocity

The first theoretical distribution of the time averaged velocities in a turbulent round jet was given by Tollmien (1926) who based his study on Prandtl's mixing length theory. Hinze (1959) and Townsend (1962) also considered the distribution of the time averaged velocities obtained according to the classical theories of Boussinesq, Prandtl, and Taylor. However, there is no analytical expression of time averaged velocities in which the mixing layer and the main region of a jet are smoothly joined to each other. It is much better to use an empirical equation which expresses the experimental values of the time averaged velocities of fluid correctly, because only the values at each point of the jet are needed for this calculation. Therefore, polynomials are used, which accurately express experimental values of time averaged fluid velocities. When we take the coordinate system as shown in Figure 1, the empirical formulae of fluid time averaged velocities are as follows:

1. Potential core

$$\overline{U} = 1, \quad \overline{V} = 0 \quad (5)$$

2. Mixing layer

$$\overline{U} = 1 - 92.6\left(\frac{\overline{r} - 0.5}{\overline{z}} + \frac{1}{13.6}\right)^2 + 340\left(\frac{\overline{r} - 0.5}{\overline{z}} + \frac{1}{13.6}\right)^3 \quad (6)$$

$$\overline{V} = -4.05\left(\frac{\overline{r} - 0.5}{\overline{z}}\right)^2 - 11.7\left(\frac{\overline{r} - 0.5}{\overline{z}}\right)^3$$

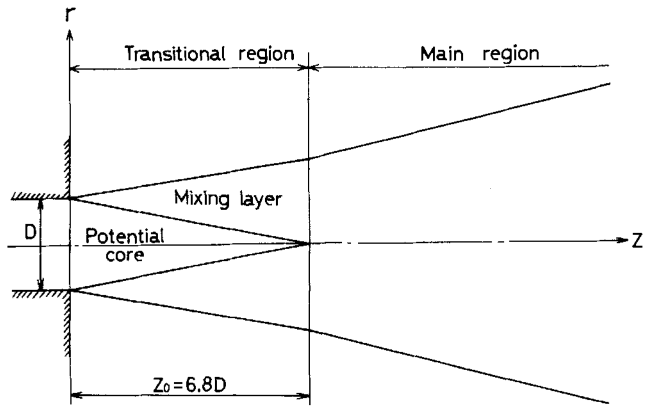

Fig. 1. The jet model.

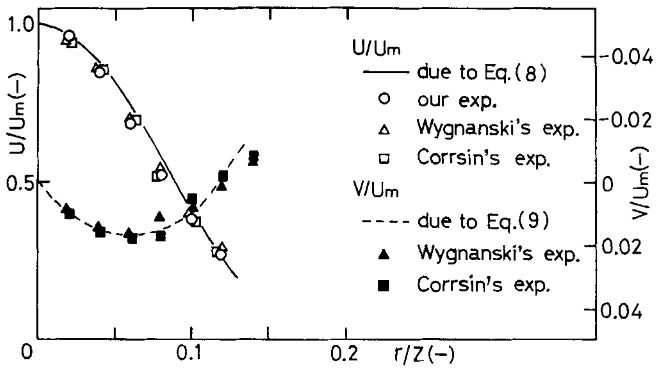

Fig. 2. Time averaged air velocity distribution.

$$+ 255\left(\frac{\bar{r} - 0.5}{\bar{z}}\right)^4 \quad (7)$$

3. Main region

$$\bar{U} = (6.8 - 630\eta^2 + 2313\eta^3)\bar{z}^{-1} \quad (8)$$

$$\bar{V} = \{3.40\eta - 472\eta^3 + 1851\eta^4\}\, \bar{z}^{-1}, \quad \eta = \bar{r}/\bar{z} \quad (9)$$

The following dimensionless variables are used:

$$\bar{z} = z/D, \quad \bar{r} = r/D, \quad \bar{U} = U/U_o,$$
$$\bar{V} = V/U_o, \quad \bar{t} = tU_o/D$$

The experimental values of Förthmann (1934), Wygnanski (1969), and the authors are plotted for these polynomials in Figure 2. As is obvious from the figure, these equations express those experimental values well.

Fluid Fluctuating Velocity

Fluctuating velocities vary their directions and sizes randomly in space and time. Liepman and Laufer (1947) Laurence (1956), and Corrsin and Uberoi (1943, 1949, 1951) obtained experimental results of fluid turbulent intensities. Both intensity distributions of the mixing layer and the main region are nearly normal. Therefore, their experimental data of intensity distributions are expressed by using normal distribution functions as follows:

1. Mixing layer

$$\langle u_z'^2 \rangle = 0.0217 \exp(-200\, \eta_1^2)\, U_o^2,$$
$$\langle u_r'^2 \rangle = 0.0103 \exp(-217\, \eta_1^2)\, U_o^2 \quad (10)$$

where $\eta_1 = (\bar{r} - 0.5)/\bar{z}$.

2. Main region

$$\langle u_z'^2 \rangle = 1.91\, U_o^2 \exp(-154\, \eta^2)/\bar{z}^2,$$
$$\langle u_r'^2 \rangle = 2.26\, U_o^2 \exp(-178\, \eta^2)/\bar{z}^2 \quad (11)$$

These empirical formulas described above agree well with the experimental values with maximum error of 4%.

Integral Scale Distributions

The Lagrangian integral time scale T_L is usually considered a measure of the longest time during which, on the average, a fluid lump persists in moving in a given direction. It seems that T_L plays an important role in the transport mechanism of the turbulent flow field. Corrsin (1963) estimated the Lagrangian time scale to be L/u'. Snyder and Lumley (1971) experimentally pointed out that Corrsin's estimate is very close to the true value. Therefore, approximate values of T_L are obtained by substitution of measured Eulerian properties into Corrsin's relation. In the present work, experimental results of L of Corrsin and Uberoi (1949), Laurence (1956), and Wygnanski (1969) are used. Considering lateral diffusion, we use the lateral integral scale for L. Empirical formulas of their results are as follows:

1. Center-line distribution of $\bar{L} = L/D$

$$\bar{L}_m = L_m/D = 0.0132\,\bar{z} \quad (12)$$

where L_m is the Eulerian integral scale at the center line of the jet.

2. Radial distribution of $\bar{L}$

$\bar{z} \leqq 4$

$$\bar{L} = \bar{L}_m[\exp\{-100[(\bar{r} + 0.45)/\bar{z}]^2\}$$
$$+ \exp\{-100[(\bar{r} - 0.45)/\bar{z}]^2\}]^{0.5} \quad (13)$$

$$4 < \bar{z} \leqq 10\; \bar{L} = \bar{L}_m \exp\{-40(\eta)^2\},$$
$$\bar{z} > 10\; \bar{L} = \bar{L}_m \exp\{-50(\eta)^2\} \quad (14)$$

CALCULATION OF PARTICLE TURBULENT DIFFUSIVITIES

Calculation of Particle Trajectories and Velocities

Since the particle is about 10^3 times as large in density as air, the particle cannot follow thoroughly the fluid motion due to the inertia. Hence, the motion of particles is not the same as that of fluid lumps. The motion of particles is treated as the motion relative to the turbulent fluid. The Lagrangian motion of particles can be calculated by Equation (4). Hinze (1959) and Hughes and Gilliland (1952) further indicated that the first and second terms on the right-hand side of Equation (4) can not be neglected only if the density of the fluid becomes comparable to or higher than the density of particles. Calculations by Tanaka and Iinoya (1971) also suggested that the Basset term is usually negligible. Since the density ratio ρ/ρ_p in this work is nearly equal to 10^{-3}, Equation (4) is simplified to the following forms:

$$\Psi\frac{d^2\bar{r}}{d\bar{t}^2} + \frac{d\bar{r}}{d\bar{t}} - \bar{v}_r = 0, \quad \Psi\frac{d^2\bar{z}}{d\bar{t}^2} + \frac{d\bar{z}}{d\bar{t}} - \bar{v}_z = 0 \quad (15)$$

The dimensionless parameter Ψ is called the Stokes number and defined as

$$\Psi = S/D, \quad S = \rho_p U_o d^2/(18\mu) \quad (16)$$

The Stokes number is the ratio of the particle stopping distance to the characteristic length. Hence, Ψ expresses the effect of particle inertia in the flow.

Instantaneous fluid velocity components in Equation (15) are written as

$$\bar{v}_z = \bar{U} + \bar{v}_z', \quad \bar{v}_r = \bar{V} + \bar{v}_r' \tag{17}$$

Fluid fluctuating velocity components $\bar{v}_z'$ and $\bar{v}_r'$ vary randomly in space and time. Fluid time averaged velocity components $\bar{U}$ and $\bar{V}$ also vary in space. It is very difficult to solve analytically the stochastic nonlinear differential Equation (15). Hence, we propose a simple model to solve Equation (15).

The particles move, surrounded by an eddy which has the lifetime T_L, and the relative motion of the particles to the fluid is caused by the interaction between fluid and particles in that eddy. T_L is defined as

$$T_L = \int_0^\infty R_L(\tau)\,d\tau \tag{18}$$

Hence, T_L is the longest time during which, on the average, a fluid lump persists in moving in a given direction. Accordingly, as in Figure 3*a*, the Lagrangian velocity correlation of the fluid $R_L = 1$ when the diffusion time $t \leqq T_L$, and that of the fluid initially surrounding the particle becomes zero when $t > T_L$ in our model. In other words, an independent eddy of the first one surrounds the particle and affects its movement during the next T_L. The concept of our model is illustrated in Figure 3*b*. The general description of the model is as follows.

The particle moves, surrounded by the eddy which has the lifetime T_{Li} when $t_i \leqq t \leqq t_i + T_{Li}$. (It is an average eddy and may be roughly considered an energy containing eddy.) If the position of particle has the coordinates ($r = r_i$, $z = z_i$) and the particle velocity components are v_{pri} and v_{pzi} at $t = t_i$, the fluid fluctuating velocities v_r' and v_z' at the same point are obtained by normal random functions whose average values are zero and variances are equal to fluid turbulent intensities shown in Equations (10) and (11), where we assume the Lagrangian fluid turbulent intensity is equal to the Eulerian one. Equations (6), (7), (8), and (9) give the time averaged fluid velocity components U and V. Substituting v_r', v_z', and Equations (6) to (9) into Equation (15), we can calculate the Lagrangian particle trajectory and the particle velocities during $T_{Li}(t_i \sim t_i + T_{Li})$ under the particle initial conditions ($r = r_i$, $z = z_i$, $dr/dt = v_{pri}$, $dz/dt = v_{pzi}$). Hence, the particle position and the particle velocity components at $t = t_i + T_{Li}$ are obtained; that is, $r = r_{i+1}$, $z = z_{i+1}$, $v_{pr} = v_{pri+1}$, $v_{pz} = v_{pzi+1}$.

Similarly, the fluid fluctuating velocities v_r' and v_z' at the next point ($r = r_{i+1}$, $z = z_{i+1}$) are obtained, and the Lagrangian particle trajectory and the particle velocities during next $T_L(t_i + T_{Li} \sim t_i + T_{Li} + T_{Li+1})$ are calculated under the next initial conditions ($r = r_{i+1}$, $z = z_{i+1}$, $dr/dt = v_{pri+1}$, $dz/dt = v_{pzi+1}$). The process noted above continues until the determined diffusion time passes. When $t = t_i + T_{Li}$, the Lagrangian velocity correlation of fluid becomes zero temporarily. Namely, this eddy disappears, and then an independent eddy appears in turn. This newly born eddy affects the particle movement for the next T_L. Hence, the fluctuating velocity of turbulent flow for the time between t_i and $t_i + T_{Li}$ keeps its direction and scalar determined at t_i during T_{Li}. Therefore, it is the average eddy that affects the particle dominantly. In our model, as may be noticed, the effect of small eddies is considered negligible based on Hinze's proposal (1972). He studied the effect of filtering these small eddies in the calculation of diffusivities. His result showed that the calculation considering the average eddy and neglecting the small ones is approximately correct. The cross-velocity correlation is also regarded as zero, according to a report by Lee

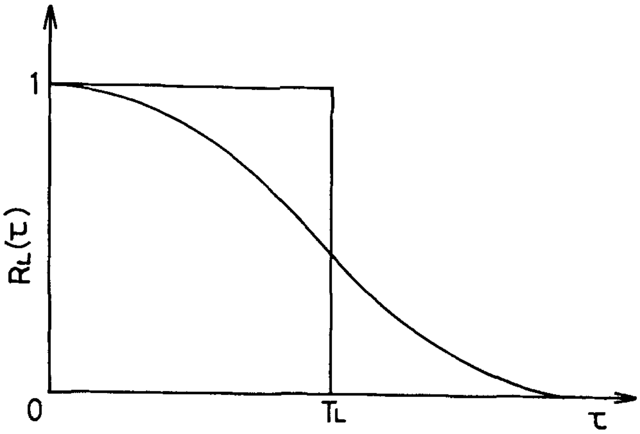

Fig. 3*a*. General idea of T_L.

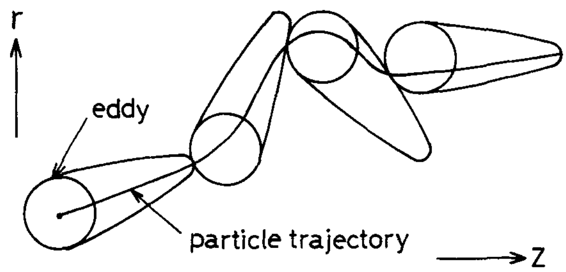

Fig. 3*b*. Particle trajectory model.

and Dukler (1976). They showed that the effect of cross-velocity correlation is negligible until the diffusion time is about 100 s. The maximum diffusion time of the present work is the time which particles flight from the nozzle exit to the furthest point [in case of $U_o = 20$ m/s, $z \doteqdot 50D$ (see Figure 13)]. The calculation of Equation (15) gives the dimensionless time $\bar{t} = tN_o/D \doteqdot 200$; then $t = \bar{t}D/U_o = 0.1$ s. Hence, the diffusion times in our case are smaller than about 0.1 s. Fluid fluctuating velocities in the numerical calculation are simulated by normal random functions. Hetsroni and Sokolov (1971) and Ribero and Witelaw (1975) gave experimental data that show that the probabilities of turbulent fluid fluctuating velocities in a round jet are nearly Gaussian. Therefore, this simulation is reasonable. The Lagrangian integral time scale can be obtained by using Equations (12), (13), and (14) and Corrsin's relation

$$T_L = \bar{L}/\sqrt{\langle \bar{u}'^2 \rangle} \tag{19}$$

where $\langle \bar{u}'^2 \rangle = \langle \bar{u}_z'^2 \rangle + \langle \bar{u}_r'^2 \rangle + \langle \bar{u}_\theta'^2 \rangle$. As Heskestad (1965) reported from his experimental result that $\langle \bar{u}_r'^2 \rangle$ is nearly equal to $\langle \bar{u}_\theta'^2 \rangle$, our calculation was performed on the assumption that $\langle \bar{u}'^2 \rangle = \langle \bar{u}_z'^2 \rangle + 2\langle \bar{u}_r'^2 \rangle$.

On the numerical calculation of Equation (15), the Runge-Kutta-Millen method was used. Here, the problem is whether or not the particles influence flow properties. In a later section it will be shown experimentally that the volumetric flow rate ratios in this work ($Q/Qa \leqq 1.99 \times 10^{-6}$) are so small that the effects on the flow of the fluid are negligible.

Particle Turbulent Diffusivity

If we calculate turbulent trajectories of a large number of particles starting from a same point by using Equation (15), a variance of particle displacements of an arbitrary point and an arbitrary time can be obtained. Since the turbulent flow field in a jet is homogeneous

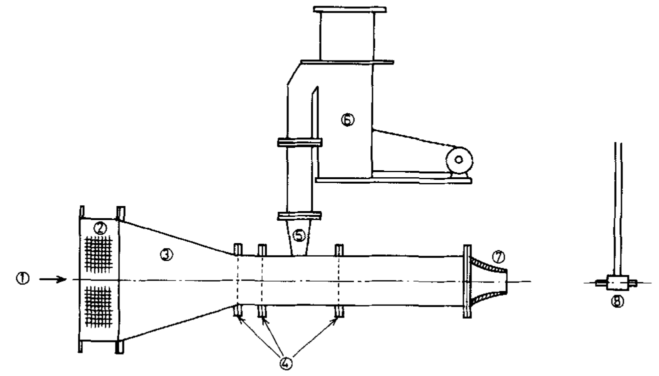

Fig. 4a. Experimental setup.

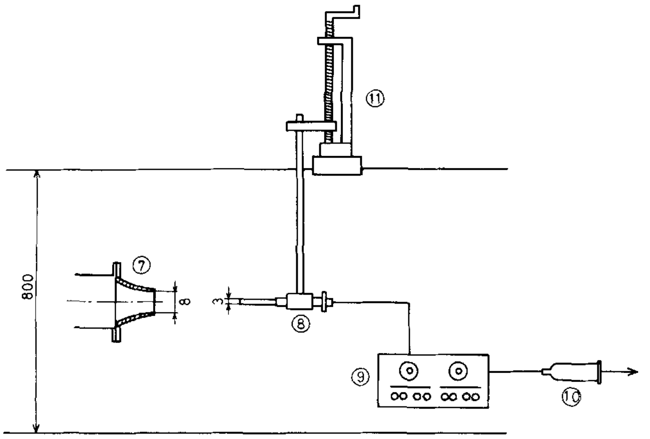

Fig. 4b. Experimental setup.

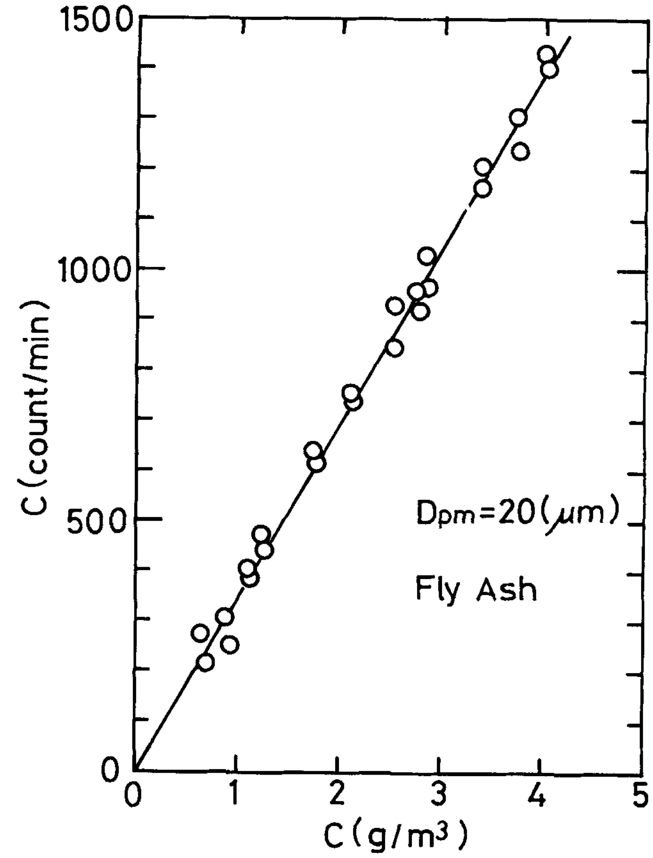

Fig. 5. Calibration curve of electric dust counter.

in time, these particles can be averaged. Substitution of the variance into Equation (3) gives the particle turbulent diffusivity.

EXPERIMENTAL

Experimental Setup

The experimental setup is shown in Figure 4. We used compressed air which led to a storage tank from which it entered the plenum chamber ① . From the plenum chamber, the air passed through a honeycomb straightener ② and converged to another plenum chamber 80 mm in diameter ③. In this plenum chamber, three 18 mesh brass grids ④ were located for producing homogeneous turbulence. The dust injector ⑤ was located on this chamber. The round jet originated from a nozzle 8 mm in diameter ⑦. The nozzle was designed, according to a scheme suggested by Smith and Wang (1944), to produce uniform velocity at the outlet. The resulting round jet exhausted into the large chamber confined between two horizontal walls extending 1.5 m downstream from the nozzle exit and 0.4 m to either side of the center line. The Reynolds numbers of the mean flow based on the nozzle diameter were 2.8×10^4 to 5.6×10^4. Velocities traverse were measured with a pitot-static probe (2 mm diameter) and Göttingen micromanometer. A brass tube calibrated in the wind tunnel was used as the pitot-static probe. The dust was fed into the injector by a smooth auto feeder (Taisei Kogyo, type CF-52) ⑥. The output of the smooth auto feeder was maintained at approximately 0.02 l/hr. Two kinds of fly ash were used for the particles. Fly ash was selected as the aerosol particle because of its spherical shape and easy dispersing. The particle size distributions were determined by the Andreasen pipet method. The computed mass mean diameters were 15 and 20 μm, and the standard deviations were 2.1 and 2.5 μm, respectively. The particle density is 2.0 g/cm³. The measurement of particle concentrations was performed with a photoelectric dust counter (Shibata Kagaku, type S-634) ⑨ calibrated by the weighing method (dust tube ⑩) (J.I.S. z8808 method). The count number of the electric dust counter (the intensity of scattered light reaching the photodetector) is plotted vs. the particle mass concentration in Figure 5. The count number is linearly related to the particle mass concentration. An isokinetics dust sampling was performed. The sampling probe ⑧ 3 mm in diameter and the pitot-static probe ⑧ were installed on the traversing equipment ⑪ . The time averaged fluid velocities and the time averaged particle concentrations were measured at seven distances from the nozzle exit, that is, z/D = 10, 20, 30, 40, 50, 60, and 70. At each cross section, data were taken at fifteen different points in each cross section.

The measurements were carried out for air and dust laden jets with volumetric flow rate ratio Q/Q_a of 4.0×10^{-7} to 2.0×10^{-6} at the nozzle exit. The issuing velocities were kept at 20 to 100 m/s for these tests. The particle concentrations C_0 (0.8 to 4 g/m³) at the nozzle exit were measured at the center line in the potential core ($\bar{z} = 4.0$). Needless to say, it is important in this work to keep C_0 constant during the measurement. The deviation of C_0 with time was within 5% as indicated in Figure 6, which is satisfactory to the condition.

Experimental Particle Turbulent Diffusivity

The equation defining the particle concentration distribution for the round jet is

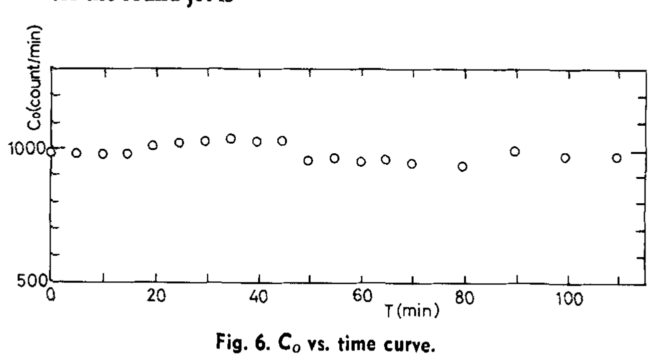

Fig. 6. C_0 vs. time curve.

$$U\frac{\partial C}{\partial z} + V\frac{\partial C}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\epsilon_{pr}\frac{\partial C}{\partial r}\right) \quad (20)$$

In Equation (20), the following assumptions are made:

1. The flow is steady, incompressible, and axisymmetric.
2. The interaction effect among particles is negligible.
3. The axial component of particle mass flux by the turbulent diffusion is negligible compared with the convective one.
4. The molecular diffusion is negligible.
5. The time averaged velocity of particles is equal to that of fluid.

The calculated results (see Figures 8 and 9) point out that assumption 5 is reasonable in the region far from the jet exit. Hence, Equation (20) is only applicable to the same region of the jet. Assuming the similarity of distribution in U, V, and C [see Hinze (1959), p. 416], Hinze gave the solution of Equation (20) as follows:

$$\epsilon_{pr} = \epsilon_m \ln\frac{U}{U_m} \Big/ \ln\frac{C}{C_m} \quad (21)$$

Hinze (1959) also gave the experimental value that $\epsilon_m = 0.013\ U_oD$ in the main region of the jet. Hinze's result of ϵ_m is applicable for the present work because the Reynolds number of the present work is nearly equal to that of Hinze. Hence, if we measure C/C_m and U/U_m, the experimental values of ϵ_{pr} can be obtained by using Equation (21). In the region where assumption 5 is not applicable, that is, the small z region, the equation defining the particle concentration distribution is

$$C\frac{\partial U_p}{\partial z} + C\frac{\partial V_p}{\partial r} + U_p\frac{\partial C}{\partial z} + V_p\frac{\partial C}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\epsilon_{pr}\frac{\partial C}{\partial r}\right) \quad (22)$$

It is very difficult to measure the high speed velocity of the small particle (U_p and V_p). Hence, if we measure C/C_m, the experimental values of ϵ_{pr} cannot be obtained by using Equation (22). Therefore, the Eulerian method based on experimental results of particle concentrations cannot give a value of ϵ_{pr} for small z. The only method that give ϵ_{pr} for small z is the Lagrangian method mentioned in a previous section.

RESULTS AND DISCUSSION

Time Averaged Velocity

The experimental distributions of time averaged air velocities of the air and dust laden jets are shown in Figure 7. One side of the distribution of time averaged velocities and dust concentrations is plotted because its symmetry on the jet axis was verified. It is clear that the volumetric flow rate ratios in this work are so small that the time averaged fluid velocity of the dust laden jet is equal to that of the single-phase jet. Hetsroni and Sokolov (1971) experimentally showed that the fluid intensity of the dust laden jet ($Q/Qa = 3.08 \times 10^{-6}$ to 7.79×10^{-6}) is nearly equal to that of the air jet. Hence, it is evident that the fluid intensity of the dust laden jet in this work is in turn nearly equal to that of a air jet, since Q/Qa is smaller than that of Hetsroni and Sokolov. Therefore, fluid properties of the single-phase jet can be used in this work. The solid line in Figure 7 calculated by using Equation (8) expresses experimental data well. It is noted that the data from the various distances do not support more than one curve and that the similarity in the time averaged fluid velocity is satisfied.

Popper et al. (1974) obtained direct information on the droplet time averaged velocity in a round turbulent air jet by means of the Laser Doppler velocimeter. Their results indicated that the time averaged velocity difference between the fluid and droplet at the jet exit was about 6% when the droplet relaxation time τ was 7.7×10^{-3} s. Since the particle relaxation time in the present work is 2.5×10^{-3} s, it is considered reasonable that U_p is nearly equal to U at the jet exit. Substituting the initial condition $U_p = U_o$ and $V_p = 0$ at the jet exit, we can calculate turbulent trajectories and velocities in the dust laden jet by using Equation (15). The calculated time averaged particle velocity was determined by averaging a large number of particle velocities at a point calculated by Equation (15). The number of particles counted, N, was about 200. The deviation of the result, which depended upon N, is inversely proportional to $\sqrt{N}$. The maximum deviation was about 7% for $N = 200$. The calculated distribution of time averaged, center-line particle velocities $\overline{U}_{pm}$ of the dust laden jet is shown in Figure 8. The larger the Stokes number is, the more slowly $\overline{U}_{pm}$ decreases with the increase of $\overline{z}$. $\overline{U}_{pm}$ is nearly equal to $\overline{U}_m$ in the region far from the points ($\overline{z} = 30$ for $\Psi = 3.4$, $\overline{z} = 42$ for $\Psi = 15$, $\overline{z} = 65$ for $\Psi = 30$). An example of the calculated distributions of U_p/U_m and U/U_m is shown in Figure 9. The region where $U_p = U$ is the coincides with the region where $U_{pm} = U_m$. An example of the calculated distributions of U_p is also summarized in Figure 10. The dimensionless

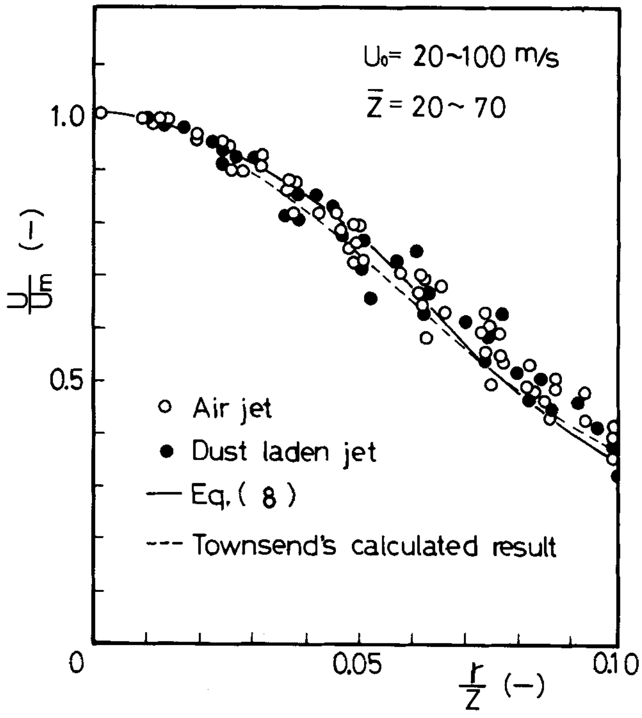

Fig. 7. Time averaged radial air velocity distribution.

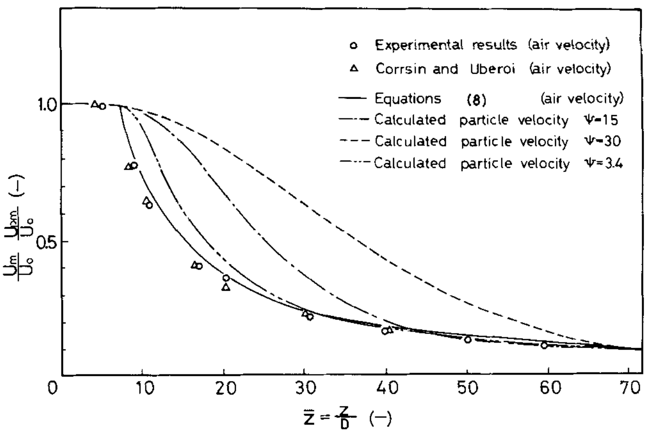

Fig. 8. Calculated time averaged axial velocity distributions of particle and air.

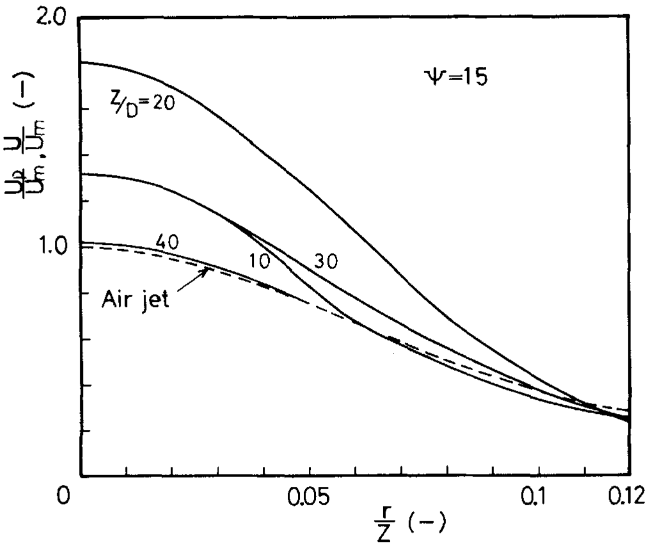

Fig. 9. Calculated time averaged radial velocity distributions of particle and air (Ψ = 15).

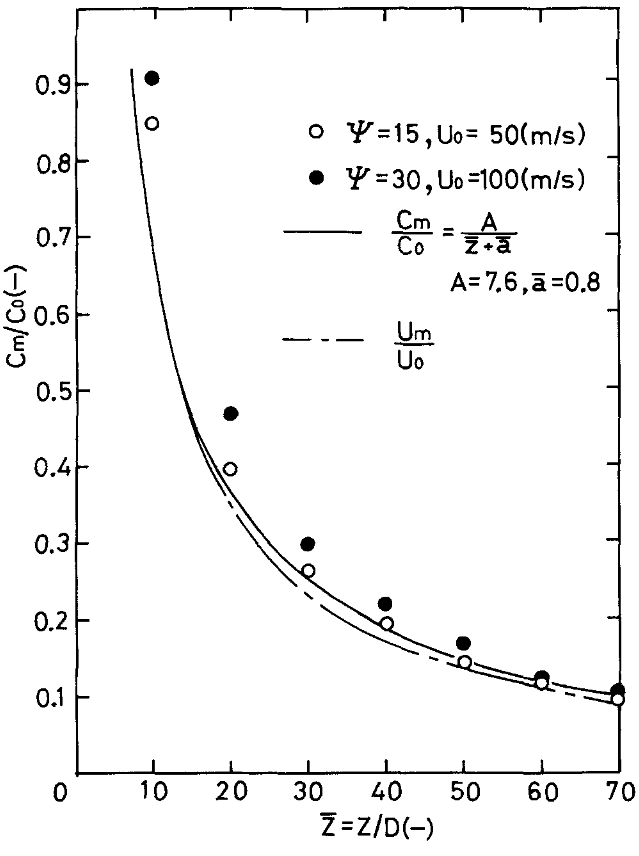

Fig. 11. Experimental results of C_m/C_o vs. z plots.

form of U_p divided by U_{pm} satisfies the similarity. The distribution of fluid velocities is wider than that of particle velocities, for particles, in spite of the effect of turbulence upstream, tend to keep their initial velocities because of their inertia. The distribution of time averaged particle velocities becomes narrower with the increase of Ψ.

Particle Concentration Distribution

The dimensionless particle concentration $\overline{C}_m$ along the center line of the jet is shown in Figure 11. The increase of Ψ makes it difficult for C_m to decrease. The line in this figure indicates the curves of $\overline{C}_m = A/(\overline{z} + \overline{a})$ which Hinze assumed from the requirement of similarity in C. We used $\overline{a} = 0.8$ which Hinze gave [see Hinze (1959), p. 426]. Experimental values of $\overline{C}_m$ can be approximately expressed by $A/\overline{z} + \overline{a}$ curve.

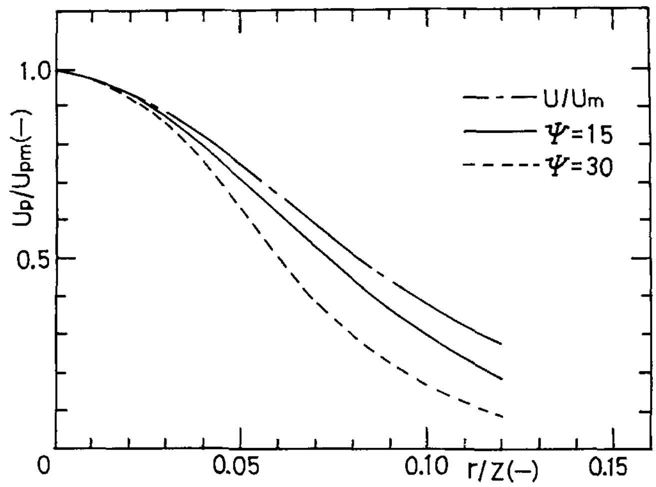

Fig. 10. Calculated results of U_p/U_{pm} vs. r/z plots.

Log-log plots of the experimental distribution of particle concentration C/C_m vs. U/U_m in the regions where $U_p = U$ are shown in Figure 12. From these values of $\ln C/C_m/\ln U/U_m$, the experimental ϵ_{pr} can be obtained by using Equation (21). It is seen that the data from the various distances downstream fall on a single curve. Hence, C/C_m may be expressed as a function of $\eta = r/z$, because U/U_m is a function of η only (see Figure 7). It is experimentally confirmed that the $U_p = U$, and the similarity of distributions in U and C, which we assumed in the foregoing section, are satisfied in the regions where $\overline{z} \geqq 30$; in the case of $\Psi = 3.4$, $\overline{z} \geqq 42$; in the case of $\Psi = 15$, $\overline{z} \geqq 65$; and in the case of $\Psi = 30$, respectively. Therefore, reasonable particle diffusivity in these regions can be obtained by using experimental values of C/C_m and U/U_m and Equation (21). The distribution of C/C_m is narrower than that of U/U_m, (compare dotted lines with solid lines in Figure 12), and it becomes narrower as Ψ increases. The narrower it is, the smaller the diffusion of particles becomes. In other words, particles tend not to diffuse easily when Ψ increases.

Particle Turbulent Diffusivity

Time averaged particle displacement $<\overline{r}_p>$ and variance of particle displacement $<(\overline{r}_p - <\overline{r}_p>)^2>$ were obtained by averaging a large number of particle trajectories ($N = 200$) at an arbitrary time t. Substitution $<(\overline{r}_p - <\overline{r}_p>)^2>$ vs. t curve into the Equation (3) gave the particle diffusivity at an arbitrary point in the jet. Calculated results of dimensionless particle diffusivity $\overline{\epsilon}_{pr} = \epsilon_{pr}/(U_oD)$ are shown in Figure 13, where parameter $\overline{r}_o$ is the particle initial dimensionless radial distance at the jet exit. Examination of this figure reveals that particle diffusivity in a jet decreases with increase of Stokes number. Since the particle inertia increases with the increase of Stokes number, the particles do not follow the fluid turbulent components in a jet. Hence, the fluctuating velocities of the particles become smaller, and $\overline{\epsilon}_{pr}$ decreases with the increase of Ψ. It is also shown that the particle diffusivity increases with $\overline{z}$ initially and eventually becomes nearly constant. One of the reasons why deviations for small $\overline{z}$ are caused from constant values of $\overline{\epsilon}_{pr}$ is that $\overline{U}_p \neq \overline{U}$. However, since $\overline{\epsilon}_m = \epsilon_m/U_oD$ is not constant in the transitional region (see Figure 1), $\overline{\epsilon}_{pr}$ in the region does not become constant if $\overline{U}_p = \overline{U}$.

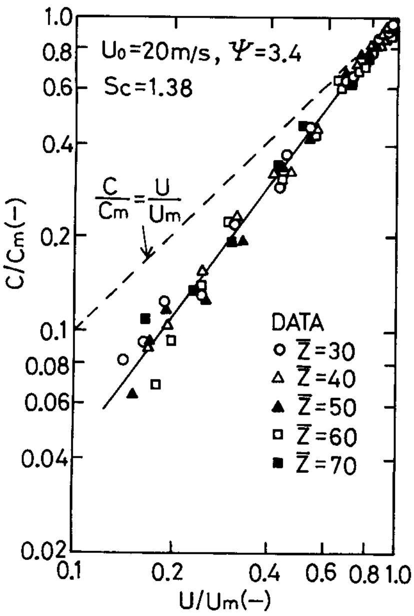

Fig. 12a. Experimental results of C/C_m vs. U/U_m plots in $U_p = U$ region ($\Psi = 3.4$).

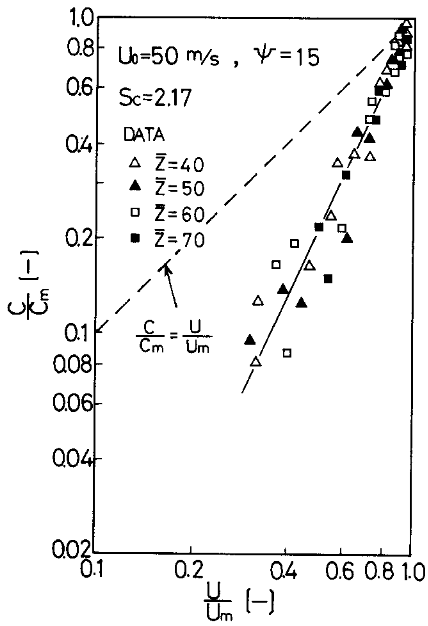

Fig. 12b. Experimental results of C/C_m vs. U/U_m plots in $U_p = U$ region ($\Psi = 15$).

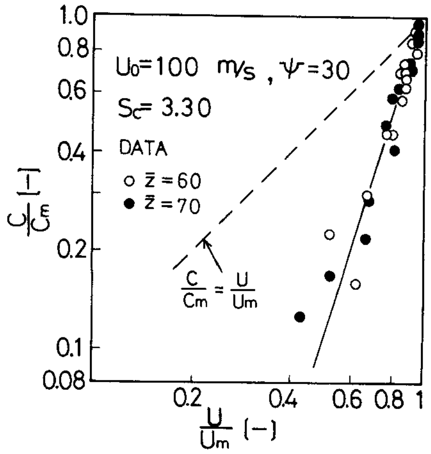

Fig. 12c. Experimental results of C/C_m vs. U/U_m plots in $U_p = U$ region ($\Psi = 30$).

The fluid turbulence is developing from the jet exit and the fully developed turbulence is established at the entrance of the main region (see Figure 1). Therefore, $\bar{\epsilon}_m$ increase from zero at the jet exit with increasing $\bar{z}$ and reaches a constant value at the entrance of the main region.

Since the particles cannot follow the fluid motion owing to their inertia, the distances from the jet exit, where values of $\bar{\epsilon}_{pr}$ reach constant, increase with the increase of Ψ. As shown in Figures 8, 9, and 13, the value of $\bar{z}$ where $\bar{\epsilon}_{pr}$ reaches constant coincides with the value of $\bar{z}$ where U_p becomes equal to U. It may be considered that particle turbulence develops fully in the region where ϵ_{pr} becomes constant. The fully developed turbulence of particles is given further downstream than that of the fluid.

The probability density distribution of the fluid fluctuating velocities for small $\bar{z}$ somewhat deviates from the Gaussian, since the fluid turbulence for small $\bar{z}$ does not fully develope. Hence, non-Gaussian behavior of the fluctuating velocities for small $\bar{z}$ may be also one of the reasons why the value of $\bar{\epsilon}_{pr}$ is not constant. The value of $\bar{\epsilon}_{pr}$ becomes larger with the increase of r_o in the transitional region of the jet. This is a reason why the radial distribution of fluid integral length scale has the maximum value at $\bar{r} = 0.5$ near the jet exit and becomes nearly constant in the cross-sectional area of the jet when $\bar{z} \geqq 20$ as shown by Laurence (1956). Values of C/C_m in Figure 12 fall on a single straight line. Hence, $\bar{\epsilon}_{pr}/\bar{\epsilon}_m$ is constant in the region where our assumptions mentioned before are valid. The gradient of ln U/U_m was obtained by the least-square fitting of experimental data. Substitution of this value into Equation (21) gives the experimental value of $\bar{\epsilon}_{pr}$. As mentioned in the preceding sections, Equation (20) is only applicable to the region of the jet where $\overline{U}_p = \overline{U}$. Hence, the experimental value of $\bar{\epsilon}_{pr}$ based on Equation (20) should be valid within the same region. As shown in Figure 13, calculated results of $\bar{\epsilon}_{pr}$ are in good agreement with the chain, the solid, and the dotted lines (the value of experimental $\bar{\epsilon}_{pr}$) in the region of the jet where $\overline{U}_p = \overline{U}$. Hence, it may be concluded that calculated results of the present work estimate true value closely in the jet. The plot of the turbulent Schmidt number in the $U_p = U$ region vs. the Stokes number, which was obtained by using the values of $\bar{\epsilon}_{pr}$ in Figure 13 and Hinze's experimental value $\bar{\epsilon}_m = 0.013$, is shown in Figure 14. Values of S_c at $\Psi = 0.016$ and $\Psi = 0$ are the results obtained by Goldschmidt and Eskinazi (1966) in the

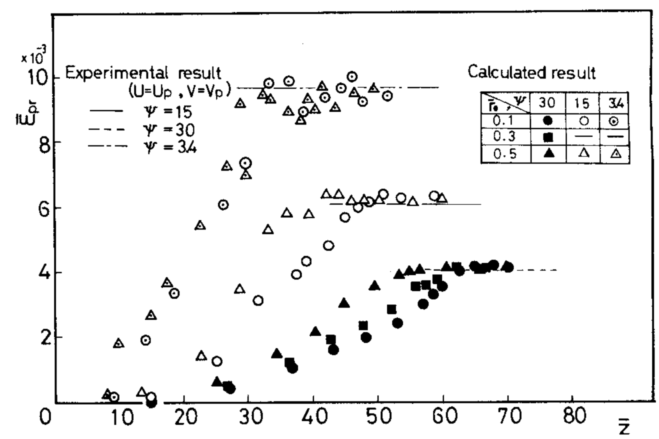

Fig. 13. Comparison of calculated and experimental particle diffusivities.

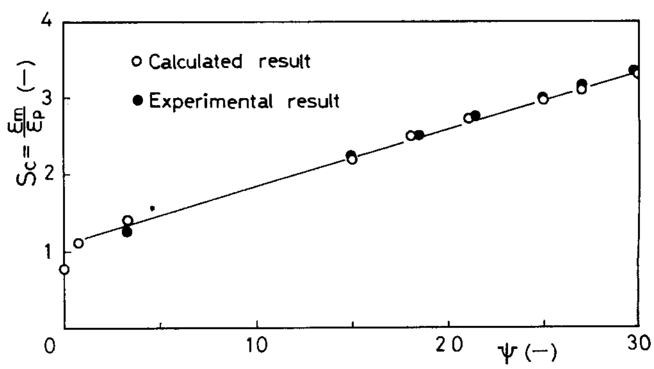

Fig. 14. Turbulent Schmidt number in $U_p = U$ region.

dust laden plane jet and by Hinze (1959), respectively. Indications of log-log plots of experimental distributions of particle concentrations and plots of $\overline{\epsilon}_{pr}$ vs. Ψ in cases of Ψ = 18, 21, 25, and 27 are omitted in this paper for the sake of simplicity. S_c increases linearly with the increases of Ψ, except in the range where Ψ is very small. These results indicate that the diffusion of particle mass becomes slower linearly than that of fluid momentum with the increase of particle inertia.

Lilly (1973) obtained the result that the larger particles were transported more effectively by turbulence than were the small particles or molecules. Lilly's flow field was a plane jet with a dust laden uniform flow at right angles to the jet center line. The velocity of the uniform flow was comparable to or larger than the jet velocity component in the same direction (lateral direction) (see Lilly's Table II). Hence, the convection of the uniform flow dominated the particle transport across the jet in the Lilly's flow field. A higher particle concentration was attained at the opposite side of the jet because the larger particles were transported faster across the jet by inertia. Lilly mistakenly thought that the particle mass flux transported by turbulent diffusion was equal to the whole particle mass flux transported by convection and turbulent diffusion. Hence, he might have concluded that the larger particle size, the smaller turbulent Schmidt number became.

ACKNOWLEDGMENT

The authors are indebted to F. Shimoda who performed some of the calculations and experimental measurements.

NOTATION

- C, C' = time averaged and fluctuating particle concentration, kg/m^3
- C_o = particle concentration at the jet exit, kg/m^3
- C_m = particle concentration along the center line of the jet, kg/m^3
- d = particle diameter, μm
- D = nozzle diameter, m
- D_B = molecular diffusivity, m^2/s
- L = Eulerian spatial integral length scale, m
- L_m = the Eulerian spatial integral length scale at the center line of the jet, m
- Q_a, Q = volumetric flow rate of the air and particles, m^3/s
- r, z = coordinates, m
- r_p = particle displacement, m
- S = particle stopping distance, m
- S_c = turbulent Schmidt number
- T_L = the Lagrangian integral time scale of fluid, s
- U, V = longitudinal and lateral time-averaged fluid velocity components, m/s
- U_m, U_{pm} = time averaged, center-line velocity of fluid and particle, m/s
- U_o = issuing fluid velocity, m/s
- U_p, V_p = longitudinal and lateral time averaged particle velocity components, m/s
- u' = Eulerian fluctuating fluid velocity, m/s
- v, v_p = Lagrangian instantaneous velocities of fluid and particle, m/s
- v' = Lagrangian fluctuating fluid velocity m/s
- z = distance from jet nozzle

Greek Letters

- ϵ = turbulent diffusivity m^2/s
- ϵ_f, ϵ_p = turbulent diffusivities of fluid lumps and particles, m^2/s
- ϵ_m = turbulent diffusivities of fluid momentum, m^2/s
- η = r/z
- μ = fluid viscosity, kg/m s
- ρ, ρ_p = mass densities of fluid and particle, kg/m^3
- τ = particle relaxation time, $\rho_p D_p^2/(18\mu)$, s
- Ψ = Stokes number, $\rho_p D_p^2 U_o/(18\mu D)$, —
- ∇ = vector differential operator known as nabla
- $< >$ = time averaged quantity

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A Modal Approach to Dynamics of Nonlinear Processes

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Approximate process dynamics of certain nonlinear systems can be estimated by elementary quadratures using a modal approach. The transient response including quadratic nonlinearities is determined by the eigenvalues, eigenvectors, and adjoint eigenvectors of the linearized system equations. The only restriction is that the dominant eigenvalue of the linearized system must be widely separated from the next slowest mode. Several process models satisfy this requirement.

The method is illustrated by application to a fourth-order model of a fluidized bed. The dynamical response is in agreement with numerical solutions to the complete model equations, including estimates of finite regions of stability.

SCOPE

The field of process dynamics has been concerned almost exclusively with the response of linear systems. This restricts the analytical understanding of the response of nonlinear processes to a small region about the steady state where a linear approximation can be assumed to be valid. There are two difficulties associated with this restriction: the analyst has no a priori knowledge of the limits of applicability of the linear analysis, and information about process behavior beyond the linear region must be obtained by direct numerical simulation. Numerical simulation can be tedious for systems of high order and/or with many parameters. Numerical simulation is particularly difficult when the system time constants are widely spaced.

This paper describes a procedure by which analytical estimates of the dynamics of nonlinear processes can be obtained for systems in which the slowest time constant is widely separated from the other time constants. The procedure is an extension of the method of modal analysis and retains the multiple time scales of the full nonlinear process. The only computational information required is obtainable from the structure of the linearized system equations.

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